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SPATIAL PRICE DISCRIMINATION WITH HETEROGENEOUS FIRMS

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ABSTRACT

In this paper we present and solve a three-stage game of entry, location, and pricing in a spatial price discrimination framework with arbitrarily many heterogeneous firms. We provide a unique characterization of all pure undominated strategy SPNE without imposing restrictions on the distribution of marginal costs or the allocation of transportation costs between firms and consumers.

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1 Introduction

In this paper we present and solve a three-stage game of entry, location, and pricing in a spatial price discrimination framework with arbitrarily many heterogeneous firms. We provide a unique characterization of all pure undominated strategy SPNE without imposing restrictions on the distribution of marginal costs or the allocation of transportation costs between firms and consumers.

We answer the following question: What determines the pattern of firm entry, location, market share, and profit in an environment in which heterogeneous firms have the ability to spatially price discriminate? In the context of the present paper, spatial price discrimination represents the ability of a firm to charge different prices to consumers at different locations in space, but does not imply a restriction that consumers cannot arbitrage away price differences. Understanding the causes of firm location is important because isolation plays a central role in determining market power. All else equal, a more isolated firm exerts greater market power over its customers.

In addition to providing what, to the best of our knowledge is the first answer to this question, the paper improves upon the empirical content of the spatial competition literature in several dimensions: (i) We do not impose restrictions on the distribution of marginal costs across firms; (ii) We do not impose a restriction on the allocation of shipping costs between firms and their customers; and (iii) We include an entry stage in which, in equilibrium, less productive firms do not enter.

Each of these contributions is potentially important for linking theory to data. For example, (i) four-digit SIC industries reviewed in Bartelsman and Doms (2000) have $85^{th} - 15^{th}$ total factor productivity ratios in the range of $2:1$ to $4:1$. Nevertheless, the new spatial competition models that incorporate firm heterogeneity all impose a restriction on the extent of permissible asymmetry between firms; see e.g. Aghion and Schankerman (2004), Syverson (2004), Alderighi and Piga (2008), and Vogel (2008). Moreover, (ii) shipping costs are substantial in a wide range of industries. Whether a supplier or a consumer incurs the cost of transportation is typically an equilibrium outcome rather than an industry-wide restriction. Nevertheless, in most spatial competition frameworks, it is assumed that either suppliers, or more often consumers incur the full cost of transportation; see e.g. Hotelling (1929), Lancaster (1979), and Salop (1979). Finally, (iii) selection on productivity appears to be an important characteristic within a wide range of industries, see e.g. Syverson (2004).

Section 2 introduces our theoretical framework. The market is represented by the unit circumference, which is populated with uniformly distributed consumers. There is a potentially large set of potential entrants with different constant marginal costs of production. These firms play a three-stage game of complete information. In the first stage, potential entrants simultaneously choose whether to enter and incur a fixed cost or exit. In the second stage, the entrants simultaneously choose their locations in the market. In the final stage, firms simultaneously set their prices, where each firm can price discriminate, potentially choosing a different price for each location in the market.

In Sections 3-5 we proceed using backward induction from the final stage to the first stage, respectively. The central testable implication of this paper, contained in Section 5, is similar to that in Vogel (2008): in equilibrium, more productive firms are more isolated—all else equal—, supply more consumers, and earn more profit. Although the current framework is applicable to fewer industries than is Vogel (2008)—as spatial price discrimination is far from a universal industry characteristic—the current paper extends the previous result in important dimensions for empirical work. In particular, there are two central differences between the result in the present paper and the previous result,[1] both of which bring theory closer to data in important respects discussed below.

First, in Vogel (2008) a unique equilibrium characterization arises only after imposing a restriction that firms incur a positive shipping cost that they cannot pass along to consumers. Without this assumption, a firm's cost of supplying a set of consumers is independent of its location. The present paper requires no such assumption. Intuitively, with price discrimination the identity of the party that pays the cost of transportation is inconsequential. The firm can always pass along this cost to the consumer; but in equilibrium it will not, since its price at each location is pinned down by the costs of its competitors. Relaxing this assumption while adding the assumption of price discrimination facilitates testing the central implication of the theory in industries, like ready-mixed concrete, with particularly good data; see e.g. Syverson (2008).

Second, a restriction is placed on the degree of permissible cost asymmetries between firms in all heterogeneous-firm, spatial competition frameworks of which we are aware. In these papers, if the set of firms violates this restriction, then one of two things occurs. If this restriction is violated in one set of papers, then firm payoffs are incorrectly specified, since at least one firm is assumed to have a negative market share so that the market shares of the remaining firms sum to a number strictly greater than one; see e.g. Aghion and Schankerman

[1] The present paper also contains results on entry. However, these are standard results; see e.g. Hopenhayn (1992), Melitz (2003), and Syverson (2004).

(2004). In the other set of papers, if all firms follow their equilibrium strategies and the set of firms violates the restriction, then highly productive firms have an incentive to deviate by undercutting less productive firms, charging a price sufficiently low that the less productive firm's market share is zero; see e.g. Vogel (2008). The present paper imposes no such restriction on the degree of permissible cost asymmetries between firms. The model with spatial price discrimination is significantly more tractable. This enables us to solve for an explicit bound on the degree of cost asymmetry between firms that have entered the market in the first stage such that an equilibrium exists in which all entrants earn positive variable profits. We show that the existence of a fixed entry cost in the first stage guarantees that no firm enters in stage one that would violate this restriction in stage two. As discussed above, in reality an average industry exhibits a wide dispersion of costs, so relaxing this assumption is crucial to bringing the theory to the data.

This is not the first paper to consider price discrimination in a spacial competition model; see e.g. Hoover (1937), Lederer and Hurter (1986), Hamilton, Thisse, and Weskamp (1989), Hamilton, MacLeod, and Thisse (1991), and MacLeod, Norman, and Thisse (1992). Building on these papers, the primary focus of which was existence of equilibria, we emphasize the determinants of isolation for arbitrarily many heterogeneous firms. This paper also contributes to a growing spatial competition literature concerned with heterogeneous firms; see e.g. Aghion and Schankerman (2004), Syverson (2004), Alderighi and Piga (2008), and Vogel (2008). Unlike Aghion and Schankerman (2004), Syverson (2004), and Alderighi and Piga (2008), the present paper considers not only endogenous prices, but also endogenous locations.

2 Setup

Consumers: The market is represented by the unit circumference—the points of which are indexed in a clockwise direction by $z \in [0, 1]$—and is populated by a unit mass of consumers who are uniformly distributed along the circumference of the circle. Each consumer is strategic and consumes one unit of a homogeneous good—buying from the lowest price source—if and only if the lowest price at which she can purchase the good, inclusive of transportation costs, is no greater than her reservation value, $v > 0$.

Firms: There is a set N containing $|N| \geq 2$ potential entrants each of which is endowed with a unique marginal cost of production $c_i \in [0, v - t/2)$.[2] Firms play a three-stage game of complete information. In the first age, *the entry stage*, firms simultaneously choose whether or not to enter and incur a fixed cost $f > 0$. Those firms that do enter move to the the second stage, *the location stage*, in which firms simultaneously choose their locations in the market. In the third stage, *the price stage*, firms simultaneously choose price schedules. Each firm i can price discriminate, choosing a price $p_i(z)$ for each location z on the circle. A firm i that is located at point η_i and sells to a consumer at point z incurs a *delivered marginal cost* of $k_i(\eta_i, z) \equiv c_i + t\|\eta_i - z\|$, where $\|\eta_i - z\|$ is the shortest arc-length separating the firm from the consumer, and $t \in (0, 2v)$ is the cost of transportation. Although all results hold whether the firm, the consumer, an any combination thereof incurs the cost of shipping, for consistency we assume throughout that firms bear the cost of transportation. Nevertheless, consumers are not restricted from taking advantage of arbitrage opportunities. If one consumer ships a good a distance d to another consumer, then the consumers incur a cost of transportation equal to td. With strategic consumers, there is a final, unmodeled stage in which consumers make their purchases.

Equilibrium concept: Throughout the paper we focus on (weakly) undominated pure strategy subgame perfect Nash Equilibria, which we refer to as equilibria. An equilibrium characterization is a vector $\{K, \mathbf{x}, \boldsymbol{\pi}\}$, where $K \subseteq N$ is the set of firms that enter the market, $\mathbf{x} \in \mathbb{R}^K$ is the vector of market shares of the entrants such that if $|K| \geq 1$ then $\sum_{i \in K} x_i = 1$ and $x_i \geq 0$ for all $i \in K$, and $\boldsymbol{\pi} \in \mathbb{R}^K$ is the vector of variable profits of the entrants.

In the following sections we use backward induction to provide the unique characterization of all equilibria in the limiting case in which the fixed cost of entry converges to zero, $f \to 0$. In addition to making the model more realistic, the assumption of a positive fixed cost of entry eliminates all equilibria that do not satisfy this unique characterization. Although it is straightforward to allow larger fixed costs, doing so would complicate notation without changing any conclusions other than restricting the set of firms that enter in equilibrium.

3 Price stage

Fix the integer number of firms in the market at $n \geq 1$, the vector of marginal costs of firms in the market at $\mathbf{c}$, and the location of all such firms $\boldsymbol{\eta}$. If $n = 1$, the monopoly charges

[2] The assumption of an upper bound on firm costs is to insure that at least one firm enters the market. The assumption that no two firms have the same marginal cost of production is for exposition only. Both assumptions could be dispensed with easily.

each location the maximum price, v, at which consumers are willing to purchase the good. In what follows, suppose that $n \geq 2$.

With Bertrand competition, heterogeneous firms, and a continuum of prices there are two standard technical issues. For simplicity, suppose that there are two firms, 1 and 2 with $c_1 < c_2$, and one location. The first technicality is that there is no pure strategy equilibrium in a game in which consumers are not strategic and the tie-breaking rule places positive probability on both firms. Fix any $p_2 \in (c_1, c_2]$. For any $p_1 < p_2$ firm 1 has an incentive to increase its price. However, if $p_1 \geq p_2$ then firm 1 has an incentive to decrease its price. To avoid this technicality we assume that consumers are strategic. With strategic consumers there exists no equilibrium in which $p_1 < p_2$ and no equilibrium in which $p_1 = p_2$ and a positive mass of consumers buys from firm 2. However, there exists an equilibrium in which $p_1 = p_2$ and consumers buy from firm 1. For this reason we assume that consumers are strategic. The second technicality is that there exist a continuum of equilibria in which firm 2 uses a weakly dominated strategy. In such an equilibrium, firm 2 charges any price $p_2 \in (c_1, c_2)$ and firm 1 charges $p_1 = p_2$. The unique equilibrium in weakly undominated strategies is $p_1 = p_2 = c_2$. For this reason we focus on equilibria in weakly undominated strategies.

With strategic consumers and Bertrand competition at each location z, the unique equilibrium price (in undominated strategies) at almost every point z, denoted $p(z)$, is

$$\begin{aligned} p(z) &= \min_{i \neq \chi(z)} k_i(\eta_i, z) \\ \text{with } \chi(z) &\equiv \arg \min_{j=1,\ldots,n} k_j\left(\eta_j, z\right). \end{aligned}$$

The firm with the lowest delivered marginal cost sells the good to consumers at point z at a price equal to the second lowest delivered marginal cost. At these prices consumers have no incentive to arbitrage: the upper bound on the price difference charged to two consumers separated by a distance of d is td, and this upper bound equals the cost that the consumers would have to incur to transport the good from one to the other.

Constructing market shares and profits: In what follows in this section we construct market shares and profits in the special case in which all firms supply a positive mass of consumers. Denote by *boundary consumer* any consumer at the boundary between the sets of consumers supplied by two firms. Suppose that no two firms are located at the same point and that firm i only sells to consumers located between its two closest neighbors, firms $i-1$ and $i+1$, where firm $i-1$ is the closest neighbor in the counterclockwise direction and firm

$i+1$ is the closest neighbor in the clockwise direction. If all firms supply a positive mass of consumers, then the boundary consumer between i and $i+1$ is a customer for whom the delivered costs of i and $i+1$ are lower than the delivered costs of any other firms.

Let $d_{i,i+1}$ and $d_{i-1,i}$ denote the distance between firm i and firm $i+1$ and between firm $i-1$ and firm i in the clockwise direction, respectively. Let $x_{i,i+1}$ and $x_{i,i-1}$ denote the distance from firm i in the clockwise direction of the boundary consumer between firm i and firm $i+1$ and in the counterclockwise direction of the boundary consumer between firm i and firm $i-1$, respectively. Firm $i+1$'s delivered marginal cost of supplying the boundary consumer between i and $i+1$ is $c_{i+1}+t\left(d_{i,i+1}-x_{i,i+1}\right)$, which equals firm i's delivered marginal cost of supplying the same consumer, $c_i+tx_{i,i+1}$. Hence,

$$x_{i,i+1} = \frac{1}{2t}\left[c_{i+1}-c_i+td_{i,i+1}\right], \text{ and similarly} \tag{1}$$

$$x_{i,i-1} = \frac{1}{2t}\left[c_{i-1}-c_i+td_{i-1,i}\right]. \tag{2}$$

Letting $x_i \equiv x_{i,i-1}+x_{i,i+1}$ denote firm i's market share, we have

$$x_i = \frac{1}{2t}\left[c_{i+1}+c_{i-1}-2c_i+tD_i\right]$$

where $D_i \equiv d_{i-1,i}+d_{i,i+1}$ denotes firm i's isolation, the distance between its two neighbors.

Normalize firm $i-1$'s location as point zero and define all other points by their distance from $i-1$ in the clockwise direction. Firm i's price at point z is determined by firm $i-1$'s delivered cost if $c_{i-1}+tz \leq c_{i+1}+t\left(D_i-z\right)$. Firm i's price at point z is determined by firm $i-1$'s delivered cost if and only if $z \leq z_i^*$, where

$$z_i^* \equiv \frac{1}{2t}\left(c_{i+1}-c_{i-1}\right)+\frac{1}{2}D_i.$$

Firm i's price at a given z is

$$p_i\left(z\right) = \begin{cases} c_{i-1}+tz & \text{if } z<z^* \\ c_{i+1}+t\left(D_i-z\right) & \text{if } z>z^* \end{cases}$$

Similarly, express the locations of the boundary consumers in terms of their distance from firm $i-1$ as $X_{i,i-1}$—which denotes the distance in the clockwise direction from firm $i-1$ of the boundary consumer between firm i and firm $i-1$—and $X_{i,i+1}$—which denotes the distance in the clockwise direction from firm $i-1$ of the boundary consumer between firm

i and firm $i+1$. We then have

$$X_{i,i-1} = \frac{1}{2}\left[d_{i-1,i} - \frac{1}{t}\left(c_{i-1} - c_i\right)\right] \tag{3}$$

$$X_{i,i+1} = d_{i-1,i} + \frac{1}{2}d_{i,i+1} + \frac{1}{2t}\left(c_{i+1} - c_i\right) \tag{4}$$

Similarly, express firm i's delivered marginal cost in terms of the distance, z, from firm $i-1$ as $k_i(z)$, where

$$k_i(z) = c_i + t\left\|d_{i-1,i} - z\right\|$$

Firm i's variable profit, π_i, can be separated into two terms, the profit it earns over the range in which its price is determined by firm $i+1$, and the profit it earns over the range in which its price is determined by firm $i-1$. These ranges are $(X_{i,i-1}, z_i^*)$ and $(z_i^*, X_{i,i+1})$ respectively, so that

$$\pi_i = \int_{X_{i,i-1}}^{z_i^*} \left[c_{i-1} + tz - k_i(z)\right] dz + \int_{z_i^*}^{X_{i,i+1}} \left[c_{i+1} + t\left(D_i - z\right) - k_i(z)\right] dz.$$

It can be shown that

$$\pi_i = t\left[\left(X_{i,i-1}\right)^2 + \left(X_{i,i+1}\right)^2 - \left(d_{i-1,i}\right)^2 - \left(z_i^*\right)^2\right] \tag{5}$$

4 Location stage

Let $K' \subseteq N$ denote the set of firms in the market. If $|K'| = 1$, then the monopoly firm is indifferent between all locations. In what follows, suppose that $|K'| \geq 2$. To crystallize ideas, we first consider a special case of the model in which each firm in the market is sufficiently productive such that it supplies a positive mass of customers in any equilibrium. Let

$$\lambda(K') \equiv \frac{1}{|K'|} + \frac{2}{t}\bar{c}(K'), \tag{6}$$

where $\bar{c}(K') \equiv \frac{1}{|K'|}\sum_{n \in K'} c_n$ is the average marginal cost of firms in the market. As shown below, in any equilibrium $\lambda(K')$ serves as an inverse measure of the toughness of competition in the market. Competition is tougher in a market with more (active) firms, holding fixed the average marginal cost (of active firms). Similarly, competition is tougher in a market with a lower average marginal cost (of active firms), holding fixed the number of (active) firms. The following Lemma provides a sufficient condition under which each firm in the market is productive enough to supply a positive mass of consumers. Under this condition, the following Lemma (i) states that an equilibrium exists and (ii) provides the unique characterization of all equilibria.[3]

Lemma 1 *Consider a location-stage subgame in which the set of the firms in the market is K, with $|K| \geq 2$, and each $n \in K$ satisfies*

$$c_n < \frac{t}{2}\lambda(K). \tag{7}$$

Then there exists an equilibrium to the location-stage subgame. In any such equilibrium, the distance between any two neighbors i and $i+1$ is given by

$$d_{i,i+1}(K) = \lambda(K) - \frac{2}{t}\left(\frac{c_i + c_{i+1}}{2}\right), \tag{8}$$

and firm i's market share and variable profit are given by

$$x_i(K) = \lambda(K) - \frac{2}{t}c_i \tag{9}$$

$$\pi_i(K) = \frac{t}{2}x_i(K)^2 \tag{10}$$

for all $i \in K$.

Unique equilibrium market shares and profits: According to Lemma 1, if each $n \in K$ satisfies Condition (7) then a firm's market share and profit are identical in all equilibria and depend on another producer's marginal cost only through its impact on the average marginal cost, $\bar{c}(K)$.

Each firm chooses its location to minimize the cost of supplying the consumers from whom it obtains the highest revenue, given the locations of all other firms. Because the cost of supplying consumers is increasing in distance, each firm locates at the center of the mass of consumers it supplies. This implies that the delivered marginal cost of supplying all boundary consumers must be equal to $\frac{t}{2}\lambda(K)$, which directly implies that firm i's market share and profit depend on its neighbors' marginal costs only through their impact on $\lambda(K)$.

This result and its economic intuition are similar to those in Proposition 1 of Vogel (2008). Nevertheless, a unique equilibrium characterization arises in Vogel (2008) only after

[3] All Proofs are relegated to the Appendix.

imposing a restriction that firms incur a positive shipping cost that they cannot pass along to consumers. Without this assumption in Vogel (2008), a firm's cost of supplying a set of consumers is independent of its location. Here, no such assumption is needed. Intuitively, with price discrimination the identity of the party that pays the cost of transportation is inconsequential. The firm can always pass along this cost to the consumer; but in equilibrium it will not, since its price at each location is pinned down by the costs of its competitors.

Permissible asymmetries: Lemma 1 provides an explicit bound—in Condition (7)—on the extent of marginal cost asymmetry under which there exists an equilibrium in which all firms supply a positive mass of consumers. According to Equation (9), if firm i violates Condition (7) and all firms follow their equilibrium strategies, then firm i's market share is zero. To understand the intuition behind Condition (7), suppose that all firms locate as prescribed by Lemma 1 and denote by $g(z) \equiv \min_{j \in K} \{k_j(z)\}$ the minimum delivered marginal cost, taken over all firms, to a consumer located at point z. Then $g(z) \leq \frac{t}{2}\lambda(K)$ for all z, since $\frac{t}{2}\lambda(K)$ is the minimum delivered marginal cost at each boundary consumer and boundary consumers face the highest prices in the market. Hence, if firm i violates Condition (7), then its revenue per sale is bounded above by its marginal cost and it is unable to earn positive variable profit. Of course, if all firms have symmetric marginal costs equal to $\bar{c}$, then all firms satisfy Condition (7) for any $|K|$.

Although this explicit bound on the extent of permissible marginal cost asymmetry represents an improvement with respect to Vogel (2008), in which no such explicit bound is derived, a goal of this paper is to provide a unique equilibrium characterization for an *arbitrary* distribution of marginal costs. It then remains to consider the case in which at least one firm violates Condition (7). The following Lemma takes a first step in this direction.

Lemma 2 *Consider an arbitrary location-stage subgame in which the non-empty set of firms in the market is $K' \subseteq N$. Then (i) there exists a unique non-empty $K(K') \equiv \left\{ i \in K' | \, c_i < \frac{t}{2}\lambda[K(K')] \right\}$; and (ii) if $K'(K) \backslash K$ is not empty, then in any equilibrium there exists at least one firm that supplies a mass zero of consumers.*

Lemma 2 provides two results for an arbitrary location-stage subgame with a non-empty set of firms K'. First, there is a unique set of firms $K \subseteq K'$, where we omit the dependence of K on K', such that (i) $c_i < \frac{t}{2}\lambda(K)$ for all $i \in K$ and (ii) for any non-empty $K'' \subseteq K' \backslash K$ there exists at least one firm $j \in K''$ such that $c_j \geq \frac{t}{2}\lambda(K \cup K'')$. Second, if K is a strict subset of K', then at least one firm in K' must supply a zero mass of consumers in any equilibrium. Looking forward, Lemma 2 insures that when the fixed cost of entry is positive and converges to zero, the unique set of firms that enter is $K(N)$. If the set of firms that enters is not $K(N)$, then there exists either (i) a firm that incurs the fixed cost and earns zero variable profit or (ii) a firm that does not incur the arbitrarily small fixed cost that would earn a discretely positive variable profit.

Although we have shown that an equilibrium exists to the location-stage subgame if all firms satisfy Condition (7), it remains to prove that an equilibrium exists if at least one firm violates this condition. The following Lemma provides an existence and characterization result for an equilibrium in a generic location-stage subgame.

Lemma 3 *Consider a location-stage subgame in which the set of the firms in the market is* $K' \subseteq N$. *If* $|K(K')| \geq 2$, *then there exists an equilibrium in which all firms* $i \in K(K')$ *choose locations and have market shares and variable profits given by Equations* (9) *and* (10), *respectively, where* $K \equiv K(K')$; *while all firms* $i \notin K(K')$ *locate at the same point as a boundary consumer and supply no consumers. If* $|K(K')| = 1$, *then there exists an equilibrium in which all firms locate together and firm* $i \in K(K')$ *supplies the entire market.*

To understand the inuition for Lemma 3, consider first whether any firm $i \notin K(K')$ has an incentive to unilaterally deviate. Given the location of all firms $j \in K(K')$, boundary consumers pay a price equal to $\frac{t}{2}\lambda[K(K')]$, and this is the minimum price paid by consumers in the market. However, by the definition of the set $K(K')$, the marginal cost of any firm $i \notin K(K')$ must be strictly greater than this price. Hence, no firm $i \notin K(K')$ can profitably supply a positive mass of consumers from any location in the market, so such a firm has no incentive to unilaterally deviate. Consider second whether any firm $j \in K(K')$ has a unilateral incentive to deviate. Given the location of all firms $i \notin K(K')$, from firm j's perspective the market is equivalent to one in which the set $K' \backslash K(K')$ is empty. Hence, firm j has no incentive to deviate according to Lemma 1.

According to Lemma 3, an equilibrium exists in any location-stage subgame. Nevertheless, Lemma 3 does not state that there is a unique characterization of all equilibria in an arbitrary subgame. In particular, Lemmas 2 and 3 do not guarantee that if $K'(K)\backslash K$ contains multiple elements, then there exists no equilibrium in which at least one firm $i \in K'(K)\backslash K$ has a positive market share. To obtain a unique equilibrium characterization, we include an entry stage with a positive fixed cost in the following Section.

5 Entry stage

In the entry stage, each firm $i \in N$ chooses whether or not to enter. If a firm chooses to enter, it incurs a fixed cost of $f > 0$ and proceeds to the location stage. Clearly, a firm chooses to enter if and only if it anticipates earning a non-negative profit. Using Lemmas 1-3, we obtain the central result of the paper in the following Proposition.

Proposition 1 *There exists an $f^* > 0$ such that for all $f < f^*$: (i) an equilibrium exists and (ii) the unique equilibrium characterization is given by $\{K(N), \mathbf{x}, \boldsymbol{\pi}\}$, where $K(N) \equiv \{i \in N | \, c_i < \lambda [K(N)]\}$ and firm i's market share and variable profit are given by Equations* (9) *and* (10)*, where $K \equiv K(N)$.*

Intuition: The existence of an equilibrium follows from Lemmas 1 and 2. According to Lemmas 1 and 2, there is a unique set of entrants, $K(N)$, such that (i) each firm $j \in K(N)$ would have a positive market share if the set of entrants were $K(N)$; and (ii) no firm $i \notin K(N)$ would have a positive market share if the set of entrants were $K(N) \cup i$. If we choose $f^* \equiv \min_{j \in K(N)} \{\pi_j [K(N)]\} / 2$, then $f^* > 0$ and $\pi_j [K(N)] > f$ for all $j \in K(N)$ and any $0 < f < f^*$. That is, each firm $j \in K(N)$ earns a positive profit from entering if the set of entrants is $K(N)$. Moreover, $\pi_i [K(N \cup i)] < f$ for any $i \notin K(N)$ and $0 < f < f^*$. That is, any firm $i \notin K(N)$ earns negative profit if it chooses to enter when the set of other entrants is $K(N)$.

Part (ii) of Proposition 1 follows from Lemmas 1-3. According to Lemma 3, an equilibrium exists for any location-stage subgame. According to Lemmas 1 and 2, if the set of firms that enters is any $K'' \neq K(N)$, then either some firm $i \in K''$ earns negative profit or some firm $j \notin K''$ would earn a positive profit if it chose to enter. Neither the existence nor the uniqueness result depend on the assumption that the fixed cost is arbitrarily small. Increasing the fixed cost merely restricts entry while requiring less parsimonious notation.

According to Proposition 1, an equilibrium exists in which at least one firm enters, the same set of firms enter in all equilibria, and each firm's market share and profit are the same across all equilibria. Moreover, this result holds for any distribution of marginal costs satisfying $c_i \in [0, v - t/2)$, where the restriction that $c_i < v - t/2$ insures that almost all consumers are supplied in all equilibria.

6 Discussion

The result in the literature most closely related to Proposition 1 is Proposition 1 in Vogel (2008). As in Vogel (2008), in this paper there is a unique characterization of all equilibria according to which more productive firms are more isolated—all else equal—, supply more consumers, and earn a higher profit. Unlike Vogel (2008), we obtain these results (*i*) for an arbitrary distribution of marginal costs, (*ii*) without imposing a restriction that firms incur a positive shipping cost that they cannot pass along to consumers, and (*iii*) while including an entry stage in which less productive firms do not enter. As discussed in the Introduction, each of these generalizations is potentially important for linking the theory to the data. In this Section we focus on the impact of the cost of transportation, the fixed cost, market toughness, and a firm's marginal cost on its isolation, market share, and profit.

Isolation: The distance between two neighbors, firm i and firm $i+1$, is greater than the average distance between firms, $1/|K|$, if and only if their average marginal cost is less than the average marginal cost of all active firms in the market. Moreover, holding the average marginal cost of active firms constant, the distance between neighbors is a strictly decreasing function of their average marginal cost. Intuitively, high-cost active firms shy away from the harsh competition of low-cost firms.

Neighbors are more isolated if there are fewer active firms in the market or if the average marginal cost of active firms in the market is greater. The impact of the number of active firms on isolation is straightforward and deserves no special mention since it is obtained in models with symmetric firms; see e.g. Salop (1979), Economides (1989), and Lancaster (1979). The impact of the average marginal cost of active firms on isolation is only obtained elsewhere, to the best of our knowledge, in Vogel (2008). If firm j's marginal cost increases and the number of firms in the market remains constant, then firm j must become less isolated. This requires that the distance between firms i and $i+1$ increases, for any $j \neq i, i+1$.

The impact of the transportation cost, t, on isolation is more complex in the current model than in Vogel (2008). As in Vogel (2008), for a fixed set of active firms, a decrease in t increases the benefit of a lower marginal cost in terms of isolation because consumers are relatively more sensitive to differences in marginal costs than differences in distances. In addition, in the current model a reduction in t also restricts entry, because profits are increasing in t for fixed locations. This provides an additional benefit of a lower marginal cost in terms of isolation. The fixed cost only affects isolation by restricting entry because it is sunk at the point at which firms choose their locations and prices.

Market share and profit: More productive firms have larger market shares and earn higher profits. In particular, a firm's market share and profit are greater than average if and only if its marginal cost is less than average, and a firm's market share and profit are decreasing in its marginal cost. The reason more productive firms supply a larger mass of consumers derives entirely from the fact that these firms are more isolated, as all firms charge the same average FOB price of $\lambda t/2$. In particular, firm i charges a minimum FOB price of $\frac{t}{2}(\lambda(K) - x_i(K))$, which equals c_i, to its boundary consumers and charges a maximum FOB price of $\frac{t}{2}(\lambda(K) + x_i(K))$ to those consumers jointly located with the firm.

In the present paper, more productive firms earn higher profits both because (i) they supply a larger mass of consumers and (ii) they charge higher absolute markups, on average. In the model, firm i's average absolute (FOB) markup is $\lambda t/2 - c_i$, which is decreasing in its marginal cost. Intuitively, productive firms set higher average absolute markups because their greater isolation provides increased monopoly power. In fact, all firms charge identical absolute (FOB) markups to consumers located z units away from their boundary customers. However, a more productive firm supplies customers located farther from its boundary customer.

The number of active firms and their average productivity affect a firm's market share and profit in the expected directions. A reduction in the cost of transportation reduces a low productivity firm's profit and market share and has an ambiguous effect on a high productivity firm's profit and market share. The direct effect of a reduction in t reduces each firm's profit. However, as noted above, reducing t both restricts entry—which increases isolation for all remaining active firms—and increases the relative return to higher productivity. An increase in the fixed cost f reduces the profit of a low productivity firm and has an ambiguous effect on a high productivity firm. The direct effect of an increase in f is to lower each active firm's profit. However, increasing f restricts entry, which increases variable profit for all firms that remain active. Clearly the direct effect wins out for the high cost firms that exit.

7 Conclusion

In this paper we presented and solved a three-stage game of entry, location, and pricing in a spatial price discrimination framework with arbitrarily many heterogeneous firms. In contrast to the spatial competition literature of which we are aware, we did not impose restrictions on the distribution of marginal costs across firms or the allocation of transportation costs between firms and consumers. Our main empirical prediction is that more productive firms are more isolated, all else equal.

One limitation of our analysis in this paper is that we have assumed that consumers are uniformly distributed through space. This is both a strong and unrealistic assumption that we made for tractability. Nonetheless, we hope that the paper provides useful insight into the determinants of firm isolation while bringing theory closer to data along a set of dimensions.

A Proofs

Proof of Lemma 1. The proof proceeds in 3 Steps.

Step 1: *Consider a location-stage subgame in which the set of firms in the market is K, with $|K| \geq 2$. If there exists an equilibrium to this subgame in which all firms supply a positive mass of consumers, then the distance between any two neighbors i and $i+1$ is given by Equation* (8) *and firm i's market share and variable profit are given by Equations* (9) *and* (10).

Proof: Suppose there exists an equilibrium to the subgame beginning in the location stage in which all firms supply a positive mass of consumers. Fix the location of all firms and consider the effect of firm i's unilateral ε-deviation towards firm $i+1$ (if $\varepsilon > 0$) or towards firm $i-1$ (if $\varepsilon < 0$). From Equations (3), (4), and (5), firm i's first-order condition for a maximum—conditional on all firms supplying a positive mass of consumers—is given by

$$d_{i,i+1}(K) = d_{i-1,i}(K) + \frac{1}{t}(c_{i-1} - c_{i+1}). \tag{11}$$

Such a location locally maximizes firm i's profits as the second-order condition is satisfied. If an equilibrium exists in which all firms supply a positive mass of consumers, then given an order of firms around the circle (i) each firm's location must satisfy Equation (11) and (ii) the sum of distances between all pairs of firms must sum to 1, i.e.

$$d_{n,1}(K) + \Sigma_{i=1}^{n-1} d_{i,i+1}(K) = 1. \tag{12}$$

Solving Equation (11) recursively yields

$$d_{i+j,i+j+1}(K) = d_{i-1,i}(K) + \frac{1}{t}(c_{i-1} + c_i - c_{i+j} - c_{i+j+1}).$$

The distance between two arbitrary neighbors as a function of the distance between firms 1 and n, where firm 1 is firm n's clockwise neighbor, is

$$d_{j,j+1}(K) = d_{n,1}(K) + \frac{1}{t}(c_n + c_1 - c_j - c_{j+1}). \tag{13}$$

Substituting Equation (13) into Equation (12) provides the solution for the distance between firm 1 and firm n

$$d_{n,1}(K) = \lambda(K) - \frac{2}{t}\left(\frac{c_n + c_1}{2}\right),$$

Substituting the solution for $d_{n,1}(K)$ into Equation (13) yields Equation (8). Given Equation (8), it is straightforward to show that market shares and variable profits are given by Equations (9) and (10).

Step 2: *In any location-stage subgame in which all $n \in K$ satisfy Condition (7), each $i \in K$ supplies a positive mass of consumers in any equilibrium.*

Proof: To obtain a contradiction, suppose that there is an equilibrium to this subgame in which a firm $i \in K$ does not supply a positive mass of consumers. The entire market must be supplied by at most the remaining $|K|-1$ firms. Moreover, $\lambda(K\backslash i) > \lambda(K)$ if and only if c_i satisfies Condition (7). Hence, the distance between at least one firm $j \in K$, $j \neq i$ and at least one of its boundary consumers is strictly greater than $\frac{1}{2}x_j(K)$, given in Equation (9). Thus, the delivered marginal cost to this boundary consumer is strictly greater than $\frac{t}{2}\lambda(K)$. Because c_i satisfies Condition (7), firm i could locate at the point at which this boundary consumer is located and supply a positive mass of consumers while earning a strictly positive variable profit, a contradiction.

Step 3: *There exists an equilibrium to any location-stage subgame in which each $n \in K$ satisfies Condition (7).*

Proof: Suppose that all firms $n \in K\backslash i$ locate as prescribed by Lemma 1. Let $g_i(z) \equiv \min_{j \neq i} k_j(z)$ denote the minimum delivered marginal cost, taken over all firms but firm i, to a consumer located at point z. Then $g_i(z)$ is continuous and $\int_{z\in\vartheta} g(z)\,dz$ denotes firm i's revenue from selling to a set ϑ of consumers. Let ϑ_i^* denote the set of consumers to whom firm i sells if firm i does not deviate from the location prescribed by Lemma 1. The lowest cost location from which to supply all $z \in \vartheta_i^*$ is the location prescribed by Lemma 1. Step 3 then follows directly from the fact that $g(z) > g(z')$ for almost all $z \in \vartheta_i^*$ and $z' \notin \vartheta_i^*$.

Lemma 1 follows directly from Steps 1-3. **QED.** ■

Proof of Lemma 2. The proof requires two preliminary steps.

Step 1: *For any $|N_0| \geq 2$ with $j \in N_0$, we have $c_j < \frac{t}{2}\lambda(N_0)$ if and only if $\lambda(N_0\backslash j) > \lambda(N_0)$.*

Proof: We have

$$\lambda(N_0\backslash j) > \lambda(N_0) \Longleftrightarrow \frac{1}{|N_0|-1} + \frac{2}{t}\overline{c}(N_0\backslash j) > \frac{1}{|N_0|} + \frac{2}{t}\left[\frac{|N_0|-1}{|N_0|}\overline{c}(N_0\backslash j) + \frac{1}{|N_0|}c_j\right]$$

which is equivalent to $c_j < \frac{t}{2}\lambda(N_0)$.

Step 2: *If $i \in K$ and $c_j < c_i$, then $j \in K$.*

Proof: To obtain a contradiction, suppose that $i \in K$, $c_j < c_i$, and $j \notin K$. If $j \notin K$, then $c_j \geq \frac{t}{2}\lambda(K \cup j)$, which implies $c_j \geq \frac{t}{2}\lambda(K)$ according to Step 1. However, $c_i \in K$ implies $c_i < \frac{t}{2}\lambda(K)$. Hence, $c_j > c_i$, a contradiction.

To obtain a contradiction to part (i), suppose that there exists a $K_1 \equiv \left\{i \in K' | \, c_i < \frac{t}{2}\lambda(K_1)\right\}$ and a $K_2 \equiv \left\{i \in K' | \, c_i < \frac{t}{2}\lambda(K_2)\right\}$ with $K_1 \neq K_2$. According to Step 2, we have either $K_1 \subset K_2$ or $K_2 \subset K_1$. Suppose that $K_1 \subset K_2$. We have

$$\lambda(K_2 \backslash V) \geq \lambda(K_2) > \lambda(K_1) \text{ for any } V \subseteq K_2 \backslash K_1 \tag{14}$$

The first inequality in Equation (14) follows from Step 1. The second inequality in Equation (14) follows from the fact that $K_1 \subset K_2$ requires the existence of at least on firm j such that $c_j \in [\lambda(K_1), \lambda(K_2))$, which implies $\lambda(K_1) < \lambda(K_2)$. Because Equation (14) holds for any $V \subseteq K_2 \backslash K_1$, it must hold for $V = K_2 \backslash K_1$. However, when $V = K_2 \backslash K_1$ Equation (14) requires $\lambda(K_1) > \lambda(K_1)$, a contradiction. Hence, there exists a unique $K \equiv \left\{i \in K' | \, c_i < \frac{t}{2}\lambda(K)\right\}$. Moreover, K is non-empty as $\frac{t}{2}\lambda(i) \equiv \frac{t}{2} + c_i > c_i$ for any firm i.

To obtain a contradiction to part (ii), suppose that $K' \backslash K$ is non-empty and there exists an equilibrium in which all firms in K' have a positive market share. According to Step 1 in the proof of Lemma 1, in any such equilibrium each firm i's market share must be given by $\lambda(K') - \frac{2}{t}c_i$. Because there exists a firm $j \notin K$, there must exist at least one firm for which $\frac{2}{t}c_i \geq \lambda(K')$. Such a firm's market share is bounded above by zero, a contradiction. **QED.**

■

Proof of Lemma 3. Consider the case in which $|K| \geq 2$. First consider an arbitrary $j \in K' \backslash K$. Given the locations of all firms $i \in K$, the maximum delivered marginal cost, taken over all $i \in K$, at any point in the market is $\frac{t}{2}\lambda(K) \leq c_j$. Hence, firm j has no incentive to deviate as it cannot earn positive variable profits from any location. Second, consider an arbitrary firm $i \in K$. Given the locations of all $j \in K' \backslash K$ and the fact that $c_j \geq \frac{t}{2}\lambda(K)$, each firm $j \in K' \backslash K$ does not impact the potential variable profits of firm i at any location that firm i chooses. Then according to Lemma 1, firm i has no incentive to deviate.

The case in which $|K| = 1$ is straightforward. No firm $j \in K' \backslash K$ can make positive variable profits locating anywhere in the market, so these firms have no incentive to deviate. Given that all firms $j \in K' \backslash K$ locate together, firm $i \in K$ earns the same variable profit no matter where it chooses to locate, so it too has no incentive to unilaterally deviate. **QED.** ■

Proof of Proposition 1. Let $f^* \equiv \min_{j \in K(N)} \{\pi_j [K(N)]/2\}$, where $f^* > 0$ follows from Lemma 1 and the definition of $K(N)$. Then $\pi_j [K(N)] > f^*$ for all $j \in K(N)$ and $\pi_i [K(N) \cup i] < 0 < f^*$ for all $i \notin K(N)$. Thus, for all $f < f^*$ an equilibrium exists in which the set of firms that enter is $K(N) \equiv \{i \in N | \, c_i < \lambda(K)\}$ and firm i's market share and variable profit are given by Equations (9) and (10).

Part (*ii*) of Proposition 1 follows from Lemmas 1-3. According to Lemma 3, an equilibrium exists for any location-stage subgame. According to Lemmas 1 and 2, if the set of firms that enters is any $K'' \neq K(N)$, then either some firm $i \in K''$ earns negative profit or some firm $j \notin K''$ would earn a positive profit if it chose to enter. ■

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